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MATDIP401

Fourth Semester B.E. Degree Examination, December 2011
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions***

1 a. If (ℓ, m, n) be the direction cosines of a line then prove that $\ell^2 + m^2 + n^2 = 1$. **(06 Marks)**

b. Find the angle between the two lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $2\ell + 2m - nm = 0$. **(07 Marks)**

c. Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$. **(07 Marks)**

2 a. Find the equation of the plane through the points $(1, -2, 2)$, $(-3, 1, -2)$ and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

b. Find the image of the point $(1, 1, 2)$ in the plane $2x + y + z - 3 = 0$. **(07 Marks)**

c. Find the shortest distance and equation between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the x – axis. **(07 Marks)**

3 a. Find the value of λ so that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are coplanar. **(06 Marks)**

b. Find $\vec{a} \cdot (\vec{b} \times \vec{c})$ and $\vec{b} \cdot (\vec{a} \times \vec{c})$, where $\vec{a} = i + j - k$, $\vec{b} = 2i - j + 2k$ and $c = 3i - j - k$. **(07 Marks)**

c. Show that the position vectors of the vertices of a triangle $2i - j + k$, $i - 3j - 5k$ and $3i - 4j - 4k$ form a right angled triangle. **(07 Marks)**

4 a. Find the unit tangent vector to the space curve $x = \cos t^2$, $y = \sin t^2$ and $z = 0$. **(06 Marks)**

b. A particle moves along a curve with parametric equations $x = t - \frac{t^3}{3}$, $y = t^2$ and $z = t + \frac{t^3}{3}$, where t is the time. Find the velocity and acceleration at any time t and also find their magnitudes at $t = 3$. **(07 Marks)**

c. Find the angle between the surfaces $x^2yz + 3xz^2 = 5$ and $x^2yz^3 = 2$ at $(1, -2, -1)$. **(07 Marks)**

5 a. Find the directional derivative of x^2yz^3 at $(1, 1, 1)$ in the direction of $i + j + 2k$. **(06 Marks)**

b. Find the constants a, b, c such that the vector $\vec{F} = (\sin y + a z)\, i + (b x \cos y + z)\, j + (x + c y)\, k$ is irrotational. **(07 Marks)**

c. Prove that $\text{dir}(\text{curl } \vec{A}) = 0$. **(07 Marks)**

6 a. Find the Laplace transform of t^n, where n is a +ve integer. **(06 Marks)**

b. Find $L[t\, e^{-2t} \cos 2t]$. **(07 Marks)**

c. Find $L\left[\frac{e^{-at} - e^{-bt}}{t}\right]$. **(07 Marks)**

7 Find the inverse Laplace transform for the following :

a. $\dfrac{s+2}{s^2+8s+25}$

b. $\dfrac{2s-1}{s^2-5s+6}$

c. $\dfrac{s}{(s^2+a^2)^2}$.

d. $\log\left(\dfrac{s+a}{s+b}\right)$.

(20 Marks)

8 a. Solve using Laplace transforms

$\dfrac{d^2y}{dt^2}-3\dfrac{dy}{dt}+2y=e^{3t}$, given that $y(0)=0$ and $y'(0)=0$. **(10 Marks)**

b. Solve the simultaneous equations using Laplace transforms $\dfrac{dx}{dt}+y=\sin t$ and $\dfrac{dy}{dt}+x=\cos t$ subject to the conditions $x(0)=2$ and $y(0)=0$. **(10 Marks)**

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